Flow Distribution Among Parallel Heated Channels

ALAN R. GRUBER and SEYMOUR C. HYMAN

Nuclear Development Corporation of America, White Plains, New York

The coolant flow distribution among parallel tubes in a nuclear reactor (or boiler or heat exchanger) can be very sensitive to variations in heat input, dimensions, etc. Analytical expressions are given for partial derivatives which measure flow variations for several situations. The utility of orifices and valves in reducing flow sensitivity is discussed. Numerical results are reported for a system using water at supercritical pressures with an eightfold expansion from inlet to outlet.

Important flow-sensitivity problems can arise in the design of a nuclear power reactor in which the coolant undergoes an appreciable decrease in density while passing through the reactor. In such a case the distribution of the coolant flow among parallel cooling tubes can become very sensitive to variations in heat input and dimensions from tube to tube, as well as to pressure variations in the headers. The term *flow sensitivity* has been coined in connection with some studies of this type on water at pressures above the critical.

Flow sensitivity is measured by the ratio of the fractional change in mass flow rate to the fractional change in average heat flux to a tube at constant pressure drop. In a similar way, terms have been created to refer to the variations in outlet fluid temperature and in maximum tube wall temperatures due to changes in heat flux, mass flow rate, tube shape or dimensions, etc. All these factors are of prime importance in reactor design. They are interrelated and depend on the physical properties of the coolant. Although the primary reference in this paper is to reactor design, exactly the same considerations and conclusions will apply to steam boilers or heat exchangers where coolants go through large temperature changes, or when a coolant has sharply temperature-dependent physical properties even though the temperature range may be small.

In a high-performance reactor for generation of useful power the rate of energy generation within a realistic size as well as the temperature level at which the energy is made available are both limited by permissible temperatures for the materials of construction. On the other hand, the economics of investment and of operation (thermodynamics) requires higher and higher temperatures. If problems of flow sensitivity lead to nonuniformity of performance among the flow channels, then only some portion of the channels can be operating at maximum wall temperatures. This could be a serious deterrent to successful operation. Flow sensitivity magnifies any other nonuniformities, such as heat flux pattern, that may be inherent in the design.

The problem of designing a nuclear power reactor so that the coolant flow pattern matches the heat generation pattern, with the coolant outlet temperature (or maximum tube-wall temperature) the same for all tubes, is made extremely difficult by the fact that most means of reactor control (to compensate for burnout of the fuel) tend to change the heat generation pattern. When employing expansible fluids as reactor coolants, the designer must be prepared either to (*a*) sacrifice potentially achievable coolant outlet temperatures by allowing for substantial differences between the maximum (design) and the average coolant outlet temperatures, or (*b*) incorporate such devices as orifices to counteract poor flow distribution and thus increase the average coolant outlet temperature at the expense of increased coolant pressure drop.

The increased sensitivity of flow rate to coolant-passage dimensions intensifies normal problems of fuel-element–manufacturing tolerances, warping and distortion of fuel elements during operation, and fouling of heat transfer surfaces.

Some typical cases in which flow sensitivity problems become important are gas-cooled reactors with large coolant-temperature rise, as studied by Addoms (*1*); reactors cooled by liquids with the coolant exit temperatures just below the boiling point, in which case a slight change in circumstances can bring about boiling in the downstream portion of the cooling tube; and reactors cooled by liquids at pressures above the critical, with the coolant bridging the critical temperature. In all these cases, the coolant flow rates and temperatures in various cooling tubes will depend sensitively on nonuniformities (most importantly, on nonuniformities in heat input) from tube to tube.

THEORETICAL DEVELOPMENT

In a nuclear reactor the heat-flux distribution along a coolant passage length may follow a symmetrical sine curve as shown in Figure 1. Specifying the value of the ratio of maximum local heat flux to average heat flux over the whole tube would then define the heat-flux pattern. The enthalpy rise of the fluid as it moves along a differential distance in the heated passage is

$$dh = \frac{4q\,dx}{GD} \tag{1}$$

Integration results in an expression for the bulk fluid enthalpy at any point along the heated passage.

$$h = h_{in} + \left[\left(\frac{q_{max}}{Q}\right)\Delta h\left(\frac{1+2\sigma}{\pi}\right)\right] \cdot \left[\cos\left(\frac{\pi\sigma}{1+2\sigma}\right) - \cos\left(\frac{\pi\sigma + \frac{x\pi}{L}}{1+2\sigma}\right)\right] \tag{2}$$

For the case of water at supercritical pressures with very large temperature differences between wall and bulk it was considered unwise to rely on the usual heat transfer film-coefficient correlations. A set of calculations was made based on a theoretical development by Goldmann (*2*). The results of the calculation showed the heat transfer to depend in a complicated fashion on the fluid properties. It was found convenient to correlate the calculated results by plotting the parameter $(qD^{0.2}/G^{0.8})$ vs. the temperature of the fluid at the wall and in the bulk. It can be shown that

$$\left(q\frac{D^{0.2}}{G^{0.8}}\right) = \left(\frac{q_{max}}{Q}\right)\left(\frac{\Delta h}{4}\right)\left(\frac{G^{0.2}D^{1.2}}{L}\right) \cdot \left(\sin\left[\left(\frac{\frac{x}{L}+\sigma}{1+2\sigma}\right)\pi\right]\right) \tag{3}$$

and so the required quantities are obtained to enable one to find the wall temperature.

It is interesting to note that the temperature profile in the fluid and at the wall at any point along the passage is determined by the parameters

$$\frac{q_{max}}{Q};\ h_i;\ h_0;\ \frac{G^{0.2}D^{1.2}}{L}$$

The calculations by Goldmann (*2*) also resulted in values of the parameter $(\tau_w\rho\ D^{0.3}/G^{1.7})$ plotted against bulk and wall temperatures. Since

$$\left(\frac{\Delta p_f}{L}\right)_{x/L} = \left(\frac{\tau_w\rho D^{0.3}}{G^{1.7}}\right)\left(\frac{4G^{1.7}}{D^{1.3}\rho}\right) \tag{4}$$

the pressure drop due to flow frictional losses can be obtained by integration

S. C. Hyman is also Associate Professor of Chemical Engineering at the City College of New York.

along the heated passage. To this are added the pressure drops due to acceleration, entrance, and exit effects. After integration

$$\frac{\Delta p_f}{G^2} = \frac{1}{G^{0.1}D^{0.1}} \cdot F\left(h_i,\ h_0,\ \frac{q_{max}}{Q},\ \frac{G^{0.2}D^{1.2}}{L}\right) \quad (5)$$

is obtained.

The entrance, exit, and acceleration losses for any particular shape of passage are of the form

$$\frac{\Delta p_{a,e,e}}{G^2} = F_1(h_i,\ h_0) \quad (6)$$

Therefore it is possible to plot $(\Delta p_f + \Delta p_{a,e,e})/G^2$ vs. the four parameters listed above. This plot does not include the effect of variations in the factor $(G^{0.1}\ D^{0.1})$.

At this stage design charts could be prepared to enable determination of pressure drops for coolant flow at any heat flux, temperature level, passage shape, or length.

If values of passage length and shape are fixed and if the shape of the heat flux pattern is fixed by fixing (q_{max}/Q), then one can plot the total pressure drop against the value of the average heat flux (corresponding then to a particular outlet temperature) at fixed values of $(G^{0.2}D^{1.2}/L)$ and inlet temperature. This enables one to obtain calculated values of the flow derivatives

$$\left(\frac{\partial \Delta p/\Delta p}{\partial Q/Q}\right)_G;\ \left(\frac{\partial \Delta p/\Delta p}{\partial G/G}\right)_Q \quad (7)$$

It could be shown that these flow derivatives are functions of the same four parameters listed previously.

From the properties of partial derivatives one obtains

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p} = -\frac{\left(\dfrac{\partial \Delta p/\Delta p}{\partial Q/Q}\right)_G}{\left(\dfrac{\partial \Delta p/\Delta p}{\partial G/G}\right)_Q} \quad (8)$$

This factor is called *flow sensitivity.*

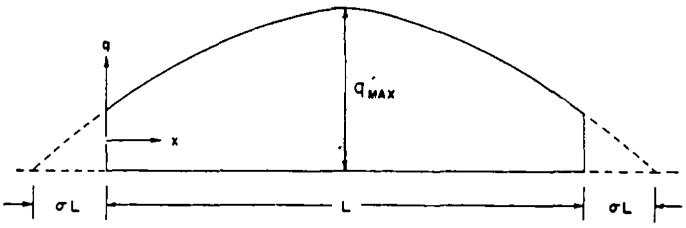
For the flow of a fluid of constant properties following standard friction factor correlations

$$\left(\frac{\partial \Delta p/\Delta p}{\partial Q/Q}\right)_G = 0;\ \left(\frac{\partial \Delta p/\Delta p}{\partial G/G}\right)_Q = 1.8;$$

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p} = 0$$

A fluid having a flow-sensitivity factor of 1.0 would maintain a fixed mixed outlet enthalpy even under conditions of varying heat input to flow passages. Such a fluid would be ideally self-regulating.

In some particular cases for real fluids, e.g., where the major effect of heat addition is a decrease in viscosity or where kinetic losses predominate, the flow-sensitivity factor may be slightly positive. Usually this factor will be negative and thus will intensify the effect of variable heat flux on outlet enthalpy.



$$q = q_{MAX}\ \sin\left[\pi\left(\frac{x+\sigma L}{1+2\sigma L}\right)\right]$$

Fig. 1. Variation of heat flux along tube.

Variation of Wall Temperature with Heat Flux

The heated-passage-wall temperature at any point is a function of the average heat flux, mass velocity, and bulk fluid temperature at this point. This assumes a prior knowledge of the form of heat flux distribution along the passage.

$$T_w = f(Q,\ G,\ T_b) \quad (9)$$

In the form of a differential equation

$$\left(\frac{dT_w}{dQ/Q}\right) = \left(\frac{\partial T_w}{\partial Q/Q}\right)_{G,T_b} + \left(\frac{\partial T_w}{\partial G/G}\right)_{Q,T_b}\left(\frac{\partial G/G}{\partial Q/Q}\right) + \left(\frac{\partial T_w}{\partial T_b}\right)_{Q,G}\left[\left(\frac{\partial T_b}{\partial Q/Q}\right)_G + \left(\frac{\partial T_b}{\partial G/G}\right)_Q\left(\frac{\partial G/G}{\partial Q/Q}\right)\right] \quad (10)$$

Define

$$e \equiv \frac{\partial h/h}{\partial T/T} \quad (11)$$

and combine with forms of Equation (1) to get

$$\left(\frac{\partial T_b}{\partial Q/Q}\right)_G = \frac{(h_b - h_{in})T_b}{e_b h_b} = -\left(\frac{\partial T_b}{\partial G/G}\right)_Q \quad (12)$$

It can be shown that, if the heat transfer coefficient correlates with the mass velocity raised to the exponent n,

$$\left(\frac{\partial T_w}{\partial G/G}\right)_{Q,T_b} = -n\left(\frac{\partial T_w}{\partial Q/Q}\right)_{G,T_b} \quad (13)$$

Substituting Equations (11), (12), and (13) into (10) with n at its usual value of 0.8 gives

$$\left(\frac{dT_w}{dQ/Q}\right) = \left(\frac{\partial T_w}{\partial Q/Q}\right)_{G,T_b} \cdot\left[0.2 + 0.8\,\frac{e_0 h_0}{T_0 \Delta h_0}\left(\frac{\partial T_0}{\partial Q/Q}\right)\right] + \left(\frac{\partial T_w}{\partial T_b}\right)_{Q,G}\left(\frac{\Delta h_b}{\Delta h_0}\right)\left(\frac{h_0}{h_b}\right)\left(\frac{T_b}{T_0}\right)\left(\frac{\partial T_0}{\partial Q/Q}\right)\frac{e_0}{e_b} \quad (14)$$

Equation (14) can be evaluated by use of the flow-sensitivity factor at constant pressure drop across the heated passage to give the effect of changing heat flux on one or some of the passages in a group operating between common headers. If the bulk values of the necessary quantities are chosen at that location in the fluid associated with the maximum wall temperature, then the calculation gives the change in the maximum temperature.

Another facet would be to inquire what the flow-sensitivity factor would have to be to maintain a constant wall temperature. This can be determined by setting Equation (14) equal to zero. The result will usually be

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{T_w} \approx 1.25$$

which could be achieved at a cost in pressure drop of

$$\left(\frac{\partial \Delta p/\Delta p}{\partial Q/Q}\right)_{T_w} = \left(\frac{\partial \Delta p/\Delta p}{\partial G/G}\right)_Q \cdot\left[\left(\frac{\partial G/G}{\partial Q/Q}\right)_{T_w} - \left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p}\right] \quad (15)$$

and a change in outlet enthalpy,

$$\left(\frac{\partial h_0}{\partial Q/Q}\right)_{T_w} = \Delta h\left[1 - \left(\frac{\partial G/G}{\partial Q/Q}\right)\right]_{T_w}$$

Effect of Variation of Hydraulic Diameter

The case in which (q_{max}/Q), h_i, and L are fixed and the h_0 is a function of T_0 will be considered.

$J \equiv (G^{0.2}\ D^{1.2}/L)$ is defined so that one may write $(\Delta p_t/G^2) = f(J,\ T_0)$ and by differentiation

$$\frac{d\left(\dfrac{\Delta p_t}{G^2}\right)}{dJ} = \left(\frac{\partial\left(\dfrac{\Delta p_t}{G^2}\right)}{\partial J}\right)_{T_0} + \left(\frac{\partial\left(\dfrac{\Delta p_t}{G^2}\right)}{\partial T_0}\right)_J \frac{dT_0}{dJ} \quad (16)$$

For the case of constant pressure drop across the heated passage $(d(\Delta p_t) = 0)$,

$$\left(\frac{\partial G/G}{\partial D/D}\right)_{\Delta p,Q} = \frac{\left(\frac{\partial\left(\frac{\Delta p}{G^2}\right)}{\partial J}\right)_{T_0} 1.2J + \left(\frac{\partial\left(\frac{\Delta p}{G^2}\right)}{\partial T_0}\right)_J \left(\frac{\partial T_0}{\partial D/D}\right)_{\Delta p,Q}}{2\left(\frac{\Delta p}{G^2}\right) + \left(\frac{\partial\left(\frac{\Delta p}{G^2}\right)}{\partial J}\right)_{T_0} 0.2J} \quad (17)$$

from Equation (1) can be obtained the relation

$$\left(\frac{\partial G/G}{\partial D/D}\right)_{\Delta p,Q} = -1 - \frac{e_0 h_0}{T_0 \Delta h}\left(\frac{\partial T_0}{\partial D/D}\right)_{\Delta p,Q} \quad (18)$$

and the simultaneous solution of Equations (17) and (18) results in

$$\left(\frac{\partial T_0}{\partial D/D}\right)_{\Delta p,Q} = \frac{1 + \left(\frac{\partial G/G}{\partial J/J}\right)_{T_0}}{-\left(\frac{\partial G/G}{\partial T_0}\right)_J - \left[1 - 0.2\left(\frac{\partial G/G}{\partial J/J}\right)_{T_0}\right]\frac{e_0 h_0}{T_0 \Delta h}} \quad (19)$$

$$\left(\frac{\partial G/G}{\partial D/D}\right)_{\Delta p,Q} = \frac{1.2\left(\frac{\partial G/G}{\partial J/J}\right)_{T_0} - \left(\frac{\partial G/G}{\partial T_0}\right)_J \frac{T_0 \Delta h}{e_0 h_0}}{\left(\frac{\partial G/G}{\partial T_0}\right)_J \frac{T_0\Delta h}{e_0 h_0} + 1 - 0.2\left(\frac{\partial G/G}{\partial J/J}\right)_{T_0}} \quad (20)$$

To complete the picture one should discover the variation of wall temperature with hydraulic diameter.

For a case of constant heat input, there can be obtained from Equations (9) and (12) the following:

$$\left(\frac{dT_w}{dD/D}\right)_{Q,\Delta p} = \left(\frac{\partial T_w}{\partial G/G}\right)_{T_b,Q}\left(\frac{\partial G/G}{\partial D/D}\right)_{Q,\Delta p} - \left(\frac{\partial T_w}{\partial T_b}\right)_{G,Q}\left(\frac{\partial G/G}{\partial D/D}\right)_{Q,\Delta p}\frac{(h_b - h_{in})T_b}{e_b h_b} \quad (21)$$

Ideal Gases in Laminar Flow

By methods similar to the method of Addoms (*1*) for laminar flow of ideal gases, with kinetic effects neglected, where

$$\mu = \alpha T^n \quad (22)$$

the flow sensitivity is

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p} = \frac{(n+1)\left(\frac{T_0}{T_i}\right)^{n+2} - (n+2)\left(\frac{T_0}{T_i}\right)^{n+1} + 1}{n\left(\frac{T_0}{T_i}\right)^{n+2} - (n+2)\left(\frac{T_0}{T_i}\right)^{n+1} + 2} \quad (23)$$

For air $n = 0.65$ and the flow shows sensitivity for all values of the temperature ratio from 1.0 to 3.7. At a temperature ratio of 3.7 the flow is unstable in the sense that any increase in heat input stops the flow.

The flow sensitivity for the kinetic losses only is

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p} = \frac{\left(\frac{T_0}{T_i}\right) - 1}{\left(\frac{T_0}{T_i}\right) - 2\frac{p_0}{p_i} + 1} \quad (24)$$

As this latter quantity is always positive, there will be some degree of heating for which the flow is moderately self-regulating.

Ideal Gases in Turbulent Flow

The flow sensitivity is

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p} = \frac{-(0.2n+1)\left(\frac{T_0}{T_i}\right)^{0.2n+2} + (0.2n+2)\left(\frac{T_0}{T_i}\right)^{0.2n+1} - 1}{(0.8 - 0.2n)\left(\frac{T_0}{T_i}\right)^{0.2n+2} + (0.2n+2)\left(\frac{T_0}{T_i}\right)^{0.2n+1} - 2.8} \quad (25)$$

Although there is never flow instability, there is always flow sensitivity unless counterbalanced by kinetic effects.

CONTROL OF FLOW SENSITIVITY BY VALVES AND ORIFICES

Control of Flow Sensitivity

The previous section of this paper gave the quantitative expressions for variation in outlet fluid temperatures and in wall temperatures resulting from nonuniformity of heat flux or net hydraulic diameter among the members of a group of flow passages. The greatest effectiveness in heat transfer to the coolant would be obtained in a design in which flow rates are so controlled as to operate each heated flow passage at its maximum safe wall temperature. It would be more practical to attempt to control flows to obtain uniform outlet fluid temperatures from the heated passages.

A flow-controlling valve sensitive to temperature at the outlet of each passage will be considered. If the outlet temperature increases, then the outlet valve is to open to permit increased flow. The sum of the pressure drops in the valve and in the heated passage must always equal the constant pressure drop available between the inlet and outlet fluid headers. Under these conditions how will the outlet temperature change when the heat flux changes with any specified sensitivity of outlet-flow-valve area to outlet temperature?

The pressure drop may be written as

$$\Delta p_v = M v_0 G_v^2 = M v_0 \frac{G_c^2 A_c^2}{A_v^2} \quad (26)$$

$$b \equiv \left(\frac{\partial \Delta p_c/\Delta p_c}{\partial Q/Q}\right)_G ; c \equiv \left(\frac{\partial \Delta p_c/\Delta p_c}{\partial G/G}\right)_Q \quad (27)$$

are defined and for the differential change in pressure drop of the heated passage one writes

$$\frac{\partial \Delta p_c}{\Delta p_c} = b\frac{\partial Q}{Q} + c\frac{\partial G}{G} \quad (28)$$

From Equation (1) at fixed D, L, h_i one gets

$$\frac{\partial G}{G} + \frac{\partial h_0}{\Delta h} = \frac{\partial Q}{Q} \quad (29)$$

$$a \equiv \frac{\partial v/v}{\partial h/h} \quad (30)$$

is defined and Equations (11), (27), (29), and (30) are substituted into Equation (28) to get

$$\Delta p_v = ae\frac{\partial T_0}{T_0}\Delta p_v + 2\frac{\partial Q}{Q}\Delta p_v - \frac{2eh_0\,\partial T_0}{T_0 \Delta h}\Delta p_v - 2\frac{\partial A_v}{A_v}\Delta p_v \quad (31)$$

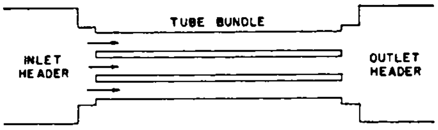


Fig. 2. Sketch of cooling tubes.

$$\partial \Delta p_c = b \frac{\partial Q}{Q} \Delta p_c + c \frac{\partial Q}{Q} \Delta p_c - \frac{ceh_0\, \partial T_0}{\Delta h T_0} \Delta p_c \quad (32)$$

For constant total pressure drop,

$$\partial \Delta p_v + \partial \Delta p_c = 0 \quad (33)$$

$$R \equiv \frac{\Delta p_v}{\Delta p_c} \quad (34)$$

is defined, and by summing Equations (31) and (32) and setting equal to zero one obtains

$$\left(\frac{\partial T_0/T_0}{\partial Q/Q}\right)_{\Delta p} = \frac{(2R + b + c)}{2RT_0\left(\frac{\partial A/A}{\partial T_0}\right)_{\Delta p} + \frac{eh_0}{\Delta h}(2R + c) - aeR} \quad (35)$$

This equation gives the variation in outlet temperature obtained with an outlet valve of given sensitivity, certain values of the flow derivatives, and the ratio of pressure drops across the control valve and in the heated passage.

A similar derivation can be made for the case of a variable-area control valve which is placed at the inlet of the heated passage but which responds to variations in outlet temperature. This gives

$$\left(\frac{\partial T_0/T_0}{\partial Q/Q}\right)_{\Delta p} = \frac{(2R + b + c)}{2RT_0\left(\frac{\partial A/A}{\partial T_0}\right)_{\Delta p} + \frac{eh_0}{\Delta h}(2R + c)} \quad (36)$$

Comparison of Equations (31) and (32) shows that the temperature response required is less for a valve at the inlet than for one at the outlet. However, the need to transmit information from the outlet location to the inlet could well be a very difficult design problem.

It follows from Equation (36) that for a fixed-area valve (orifice) at the inlet the relationship is

$$\left(\frac{\partial T_0/T_0}{\partial Q/Q}\right)_{\Delta p} = \frac{(2R + b + c)}{\frac{e_0 h_0}{\Delta h}(2R + c)} \quad (37)$$

As the value of R defined in Equation (34) is increased, the outlet temperature sensitivity decreases. This is to be expected when the flow through the valve rather than through the heated passage provides the controlling pressure drop. It may be of interest to compare the case of a fixed outlet valve to a fixed inlet valve by using Equation (35) for an orifice of $(\partial A/A/\partial T_0)_{\Delta p} = 0$ and Equation (37). One obtains

$$\frac{R_i}{R_0} = \frac{2b - a\frac{\Delta h}{h_0}(b + c)}{2b + 2a\frac{\Delta h}{h_0}R_0} \quad (38)$$

As this indicates a ratio of less than one, it appears that an inlet orifice will always be less expensive in pressure drop than an outlet orifice of equal effectiveness. In fact it could be shown that an outlet orifice of very large pressure drop is only moderately better than no orifice at all.

Control at Less than Full Load

The value of fixed orifice control at less than design load for the reactor will be considered. With operation presumed at the same inlet and outlet temperatures at all loads, G must vary in proportion to Q (the reactor heat load). The pressure drop through the orifice will vary with G^2 while pressure drop through the heated passage will vary with $G^{1.8}$ approximately. At less than design load, then, the ratio R will be somewhat less than its design value. This change is predictable, small, and easily allowed for in the design. However, it means that the orifice must

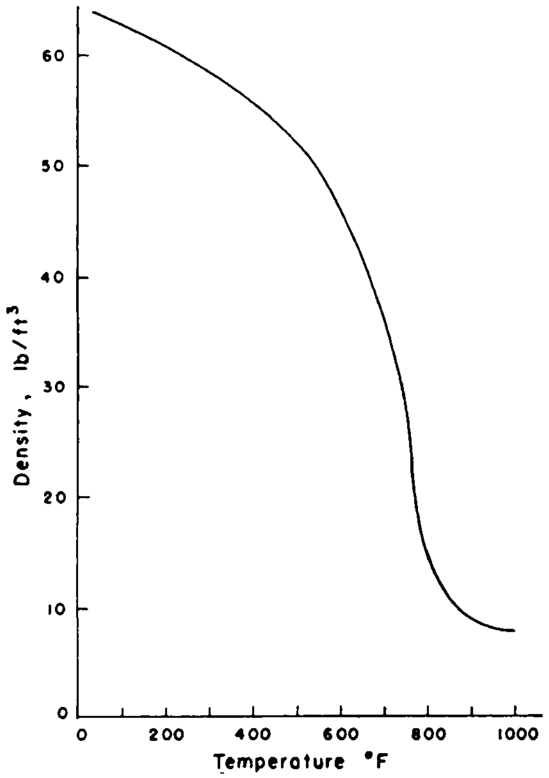


Fig. 3. Density of water at 5,000 lb./sq. in.

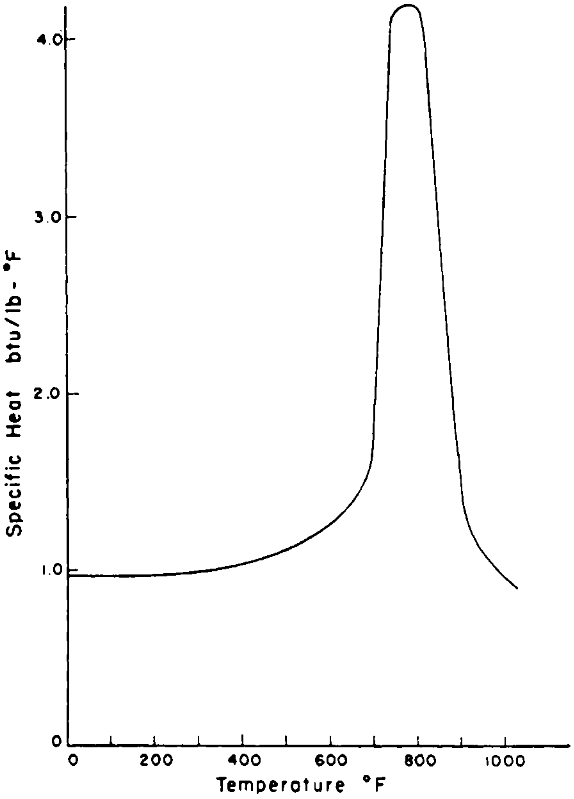


Fig. 4. Specific heat of water at 5,000 lb./sq. in.

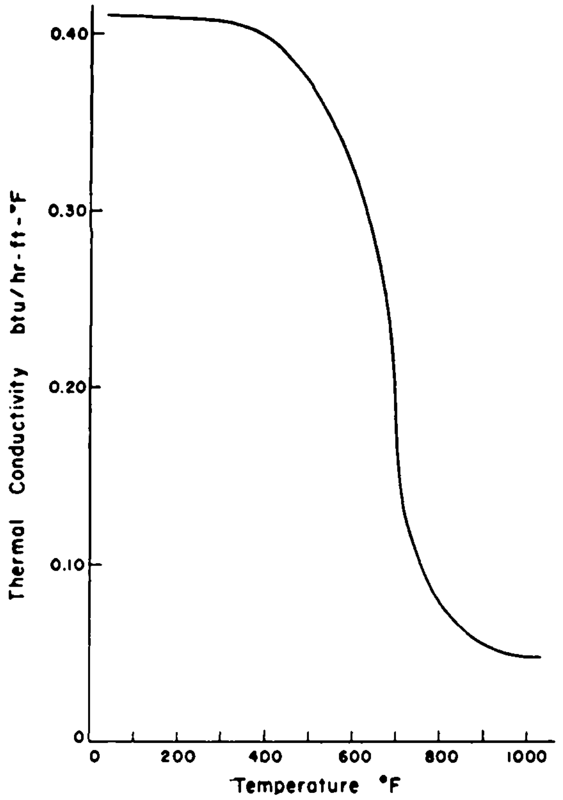


Fig. 5. Thermal conductivity of water at 5,000 lb./sq.in.

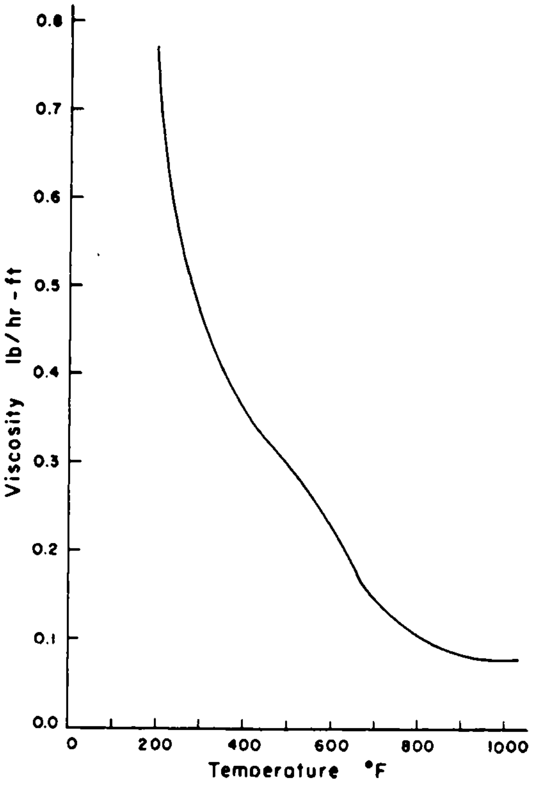


Fig. 6. Viscosity of water at 5,000 lb./sq. in.

be made to dissipate a slightly larger pressure drop at design loads in order to retain control action at reduced loads.

Manufacturing Tolerances for Orifices

If Equation (37) for a variable-area inlet valve is rewritten for the case of fixed heat input ($\partial Q = 0$), under the condition that all area variations are due to manufacturing tolerances, one gets for an outlet temperature sensitivity in this case

$$\left(\frac{\partial T_0}{\partial A/A}\right)_{\Delta p, Q} = -\frac{2RT_0\Delta h}{eh_0(2R + c)} \quad (39)$$

For a sharp-edged orifice one can write

$$\Delta p_r = \frac{G_r^2}{2k^2 g\rho} = \frac{G_r^2 A_c^2}{A_r^2 2k^2 g\rho}\frac{\Delta p_c R}{r} \quad (40)$$

For an orifice plate with N equal-size holes the area of the orifice is

$$A_v = \frac{\pi}{4} D_v^2 N \quad (41)$$

which combines with Equations (39) and (40) to give for the tolerance on orifice diameter

$$\frac{\partial D_v}{D_v} = -\frac{eh_0(2R + c)}{4T_0 R\Delta h}(\partial T_0) \quad (42)$$

where the value of the diameter is obtained from

$$D_v = 0.949\left(\frac{G_c A_c}{kN}\right)^{1/2}\left(\frac{r}{\rho g \Delta p_c R}\right)^{1/4} \quad (43)$$

The tolerance given by Equation (42) assumes that all hole diameters in a multiple-hole orifice plate deviate in the same direction.

CALCULATIONS FOR A SUPERCRITICAL WATER BOILER

The material presented in the previous sections of this paper will be used for the discussion of a particular case. This case is typical of the results obtained in a series of calculations on water at supercritical pressures, heated in parallel tubes of normally identical heat input, with the tubes between common headers. It might be noted that the temperatures and pressures of the example are similar to those of a large central-station supercritical-pressure boiler now being built for a utility company (*3*). This boiler has one-pass flow instead of recirculation as in most large boilers and would be subject to problems of flow sensitivity. Dimensions, flow rates, and heat flux are assumed arbitrarily at reasonable values and do not reflect the values for any industrial power plant or any nuclear reactor.

The Typical Case

A parallel set of cooling tubes, as sketched in Figure 2, with all the tubes nominally having identical heat input and flow conditions as given in Table 1 will be considered. The physical arrangement is pictured as several tubes clustered in a bundle with bundles arranged between large headers. The headers are assumed to be so large as to allow calculations on the basis of loss of one half a velocity head at tube entrance and one velocity head at tube outlet.

TABLE 1

p	5,000	lb./sq. in.
T_i	515	°F.
T_0	1,150	°F.
Q	3×10^5	B.t.u./(hr.)(sq. ft.)
G	1.8×10^6	lb./(hr.)(sq. ft.)
D	0.0416	ft.
L	62	ft.

It is assumed that the pressure drop between the headers is very small compared to 5,000 lb./sq. in. The relevant properties of water at 5,000 lb./sq. in. are shown in Figures 3 to 6. The density (Figure 3) and specific heat (Figure 4) are from Keenan and Keyes (*4*); the viscosity and thermal conductivity are extrapolated from the sparse data at lower pressures. It should be noted that the water undergoes eightfold expansion from 515° to 1,150°F.

For the example of Table 1, it is found that the maximum temperature of the tube heating surface is 1,370°F. and the pressure drop is 44.9 lb./sq. in. Heat transfer coefficients and friction factors were evaluated according to calculations of Goldmann (*2*). For present purposes, the Goldmann predictions give results substantially the same as if the method of Deissler (*5*) were used or if the usual isothermal turbulent flow formulas were used with properties based on mean film temperatures.

Flow-sensitivity Derivatives

The calculations for the example of Table 1 gave numerical values for the quantities of Equations (7) and (8) as follows:

$$\left(\frac{\partial \Delta p/\Delta p}{\partial Q/Q}\right)_G = 1.50;$$

$$\left(\frac{\partial \Delta p/\Delta p}{\partial G/G}\right)_Q = 0.40;$$

$$\left(\frac{\partial G/G}{\partial Q/Q}\right)_{\Delta p} = -3.75$$

This latter quantity is the *flow sensitivity*.

From this flow sensitivity and Equation (1) can be calculated for this case.

$$\left(\frac{\partial h_0/\Delta h}{\partial Q/Q}\right)_{\Delta p} = 4.75;$$

$$\left(\frac{\partial T_0}{\partial Q/Q}\right)_{\Delta p} = 6{,}700°\text{F}.$$

If one of the group of tubes between the common headers should receive 1% more heat input than its neighbors, then the fluid outlet of that tube would run 67°F. hotter than the others. It is of greater importance to evaluate the probable effect on wall temperature. In some designs for maximum performance any appreciable increase in wall temperature could seriously limit the life of the machine. Numerical evaluation of Equation (14) gives

$$\left(\frac{dT_w}{dQ/Q}\right)_{\Delta p} = 8{,}000°\text{F}.$$

This tube of 1% higher heat input is also running at 80°F. increased wall temperature and 3.75% reduced flow. These figures are impressive when one considers that many significant heat transfer as well as nuclear variables cannot be predicted or even measured to better than several per cent accuracy.

The designer will also be interested in the effect of nonuniformities in tube diameters. The flow sensitivities in this regard are indicated by the calculated values of the derivatives of Equations (19), (20), and (21):

$$\left(\frac{\partial T_0}{\partial D/D}\right)_{\Delta p,Q} = -10{,}300°\text{F};$$

$$\left(\frac{\partial G/G}{\partial D/D}\right)_{\Delta p,Q} = 6.3$$

$$\left(\frac{dT_w}{dD/D}\right)_{\Delta p,Q} = -11{,}500°\text{F}.$$

One can calculate the sensitivity derivatives reflecting the influence of header pressures on fluid and wall temperatures.

$$\left(\frac{\partial T_0}{\partial \Delta p/\Delta p}\right)_Q = -3{,}400°\text{F}.;$$

$$\left(\frac{\partial T_w}{\partial \Delta p/\Delta p}\right)_Q = -3{,}850°\text{F}.$$

Reduction of Flow Sensitivity

The flow and temperature changes cited above for small changes in heat input, tube diameter, and pressure drop can be reduced appreciably by modifying the design to incorporate orifices at the tube inlets.

Probably the best combination of effectiveness and practicality is to build in a fixed-area orifice at the inlet of each tube. Comparison with variable valves and orifices at the tube outlet were made earlier in this paper. It does not require very large pressure expenditure across the orifice to bring about marked reduction in sensitivity. Table 2 shows how the sensitivity to nonuniformity in heat input can be reduced by orifices.

It is seen that even the case of orifice pressure drop just equal to tube pressure drop makes the flow in the tube nearly independent of the power variation. The infinite orificing would make the flow constant, regardless of power, as for a constant-property fluid.

From Equation (42) one can calculate that a tolerance of 1% on the diameter of the controlling orifice will give a variation of 20°F. in the outlet fluid and the wall temperatures. It would be practical to hold tolerances to much smaller limits.

The sensitivity of the flow to variations in hydraulic diameter among the heated passages is also greatly reduced by the use of an inlet orifice.

Other Design Methods

Another possibility for decreasing flow sensitivity is to divide the tubes into several shorter lengths, with intermediate flow-mixing headers to permit equalization of temperatures and pressures across the tube bank. This would prevent the accumulation of effects along the entire length of an excess-powered tube. The major difficulty in actual practice is achieving good lateral temperature equalization in small headers. However, pressure equalization in intermediate headers even without appreciable mixing will substantially reduce sensitivity.

When heat-generation patterns as well as flow sensitivities are known, it is possible to calculate net mixed temperatures as well as the effects of change in operating levels of power or temperature. Both of the procedures have been investigated in studies subsequent to those reported herein.

CONCLUSION

The usual degree of sensitivity demonstrated in this example is primarily inherent in the nature of the fluid. However, engineering design can counteract most of the bad effects if the system is carefully studied. Accurate fluid friction and heat transfer data are prerequisites to successful design of power plants using sensitive fluids.

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TABLE 2

Orifice pressure drop / tube pressure drop R	Increase in water outlet temperature (°F.) for tube with 1% excess power [Equation (37)]	Increase in maximum heating-surface temperature (°F.) for tube with 1% excess power [Equations (37) and (14)]
0*	67	80
0.5	29	35
1	23	28
2	19	23
10	15	18
∞	14	16

*No orifice.

NOTATION

- A = area, sq. ft.
- a = symbol for derivative as defined in the text
- b = symbol for derivative as defined in the text
- c = symbol for derivative as defined in the text
- D = equivalent diameter of heated passage or tube
- e = symbol for derivative as defined in the text
- G = mass velocity, lb. mass/(hr.)(sq. ft.)

g = conversion factor, 32.2 lb. mass-ft./lb. force-sec.^{2}
h = enthalpy, B.t.u./lb. mass
Δh = difference in enthalpy between two locations, B.t.u./lb. mass
J = symbol for parameter as defined in the text
k = orifice coefficient
L = length of heated passage, ft.
M = constant of proportionality
N = number of equal-size holes in each orifice plate
n = exponent on mass velocity term or on temperature
p = pressure, lb. force/sq. ft.
Δp = difference in pressure between two locations, lb. force/sq. ft.
Q = average heat flux along heated passage, B.t.u./(hr.)(sq. ft.)
q = local heat flux, B.t.u./(hr.)(sq. ft.)
R = ratio of control valve pressure drop to fluid passage pressure drop
r = fractional permanent pressure loss across an orifice
T = temperature, °F.
v = specific volume, cu. ft./lb. mass
x = distance from passage inlet, ft.
α = coefficient in Equation (22)
ρ = density, lb. mass/cu. ft.
τ = shear stress, lb. force/sq. ft.
σ = fraction of cut-off in sine distribution
μ = viscosity, lb. mass/(ft.)(hr.)

Subscripts

a = acceleration
b = bulk
c = heated passage
e = entrance, exit
f = friction
i = inlet
max = maximum
o = outlet
t = total
v = valve or orifice
w = wall

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Light Transmittance as a Measure of Interfacial Area in Liquid-liquid Dispersions

VIRGIL G. TRICE, JR., and WALTON A. RODGER

Argonne National Laboratory, Lemont, Illinois

Interfacial area in liquid-liquid systems has been measured photographically. Precision and accuracy of the method have been shown to be better than 10%. To avoid tedium of counting drops, a simple light probe of easily reproducible design has been developed to measure the light transmission through the dispersions formed. A correlation of light transmittance with interfacial area is presented and its usefulness and limitations are discussed.

Frequently workers concerned with dispersions of immiscible liquids desire knowledge of the interfacial area of these systems. Unfortunately it is generally difficult or impossible to determine the interfacial area of liquid-liquid dispersions. In the few cases where interfacial areas have been determined, most investigators have relied on photographic techniques.

In recent years the phenomenon of light scattering by small particles has been the subject of a number of investigations (*14*); however, very little work has been reported in the literature concerning light scattering as a measure of particle size in liquid-liquid dispersions. Undoubtedly a major deterrent to such investigations has been the problem of measuring the interfacial area of rapidly coalescing disperse systems by other than photographic methods.

In this paper a photographic technique for measuring interfacial area is described.